Surds

What are surds?

Surds are mathematical expressions that involve square roots of numbers that are not perfect squares. In other words, a surd is an expression that cannot be simplified to remove a square root. For example, √2 and √5 are surds because they cannot be simplified further.

Simplifying surds:

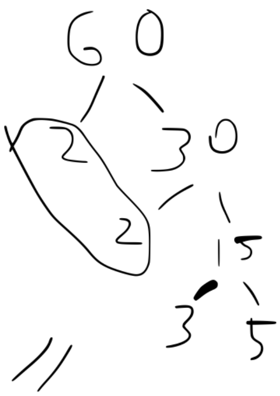
To simplify a surd from a big number on the inside of the root to a simple number you need to find the prime factorization of the number like this:

$$\sqrt{60}$$

$$\sqrt{60}$$

$$= 2\sqrt{3 \times 5}$$

$$= 2\sqrt{15} \; //$$



After finding the prime factorization you can group common prime factors and take them outside the surd. The remaining numbers with no pairs can be multiplied inside the root and that will produce the simplified version of the surd. If there are no pairs from the prime factorization that means the surd is already simplified to its simplest form. Here is another example:

$$\sqrt{48}$$

$$= 2 \times 2\sqrt{3}$$

$$= 4\sqrt{3} \; //$$

In this example, there were two pairs from the prime factorization. For this situation you must multiply the pair outside the root and leave the remaining prime factors inside the root.

Rule 1: Adding roots and subtracting surds

When adding or subtracting surds you must first simplify the surds so that they both reach common values inside the root. Once that is done you can combine like terms and operate with the coefficients of the roots. Here is an example of adding and subtracting surds:

$$\sqrt{48} + \sqrt{12} = ?$$
$$\sqrt{48} = 4\sqrt{3}, \sqrt{12} = 2\sqrt{3}$$
$$4\sqrt{3} + 2\sqrt{3} = 6\sqrt{3} //$$
$$\sqrt{48} - \sqrt{12} = ?$$
$$4\sqrt{3} - 2\sqrt{3} = 2\sqrt{3} //$$

However, in some cases, the simplified roots still do not have common values inside the root, hence they cannot be combined or operated on so they must be left in the expression respectively. Here is an example:

$$\sqrt{48} - \sqrt{18}$$
$$\sqrt{48} = 4\sqrt{3}, \sqrt{18} = 3\sqrt{2}$$
$$4\sqrt{3} - 3\sqrt{2} //$$

Rule 2: Multiplying surds

When multiplying surds, you first multiply the inside values from the roots, then you multiply the coefficients of the two surds. Then you find the prime factorization of the inside value of the root to try and simplify it. If you find pairs, multiply those values to the already existing coefficients of the root and leave the remaining prime factors to multiply in the root. Here is an example:

$$4\sqrt{3} \times 3\sqrt{6} = ?$$

$$4(3) = 12,\ 3(6) = 18$$

$$12\sqrt{18}$$

$$18 \to 2,\ 9 \to 3,\ 3$$

$$12(3) = 36$$

$$36\sqrt{2}$$

In this example, the surds were already simplified to their simplest form before getting multiplied so we can just go ahead and multiply the coefficients and the roots straight away. After combining two values, you need to do the prime factorization of the product in the root to see if it can be simplified further. In this case we had a single pair of 3 so you multiply the coefficient of the surd by 3 and leave the remaining prime factor in the root.

Rule 3: Dividing surds

When dividing surds you can take the root and wrap it around the whole expression. It can only be wrapped around two surd values if they are being directly divided. Then you can simplify the expression by dividing the two integers in the fraction to produce a singular surd. Then you simplify that surd to its simplest form. Here is an example:

$$\frac{\sqrt{6}}{\sqrt{3}} = \sqrt{\frac{6}{3}} = \sqrt{2}$$

$$\frac{12\sqrt{21}}{3\sqrt{7}} = \frac{12}{3} \times \sqrt{\frac{21}{7}}$$

$$= 4\sqrt{3}$$

Rule 4: Rationalizing the denominator

When rationalizing, you should first make sure that all the surds in the expression are at its simplest form since its easier to work with smaller values. There are two ways to rationalize the denominator of a surd, first way is if there is only a surd present in the denominator. For this situation you only multiply the same surd to both the numerator and denominator to make the denominator an integer. You can then do simple division to further simplify the expression if it can be simplified. The second situation is when there is a surd with another integer involved in the expression. In this situation you want to multiply the numerator and denominator by the same expression with the opposite expression to produce an integer in the denominator's position where you can simplify the fraction further again. Here is an example:

$$\frac{\sqrt{2}+3}{\sqrt{2}-1}$$

$$\Rightarrow \frac{(\sqrt{2}+3)(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)}$$

$$\Rightarrow \frac{2+\sqrt{2}+3\sqrt{2}+3}{2+\sqrt{2}-\sqrt{2}-1}$$

$$= 5+4\sqrt{2}$$

Here is another example:

$$\frac{15}{\sqrt{3}}$$

$$\Rightarrow \frac{15(\sqrt{3})}{\sqrt{3}(\sqrt{3})}$$

$$\Rightarrow \frac{15\sqrt{3}}{3} = 5\sqrt{3}$$

Rule 5: Multiplying surd expressions

When multiplying surd expressions, you follow the same rules as expanding any brackets. You then have to simplify the final expression to its simplest form. Here is an example:

$$\left(\sqrt{3}+1\right)\left(\sqrt{5}+2\right)=\sqrt{15}+2\sqrt{3}+\sqrt{5}+2$$

In this example there are no like terms and the surds are all in their simplest form, so the final answer is to be kept like this. Here is a different example:

$$\left(\sqrt{2}+1\right)\left(\sqrt{2}+2\right)=2+2\sqrt{2}+\sqrt{2}+2$$

$$=4+3\sqrt{2}$$

In this example there were like terms, so I simplified it further. Here is a different example:

$$\frac{\left(\left(\sqrt{2}+5\right)\left(3-\sqrt{5}\right)\right)}{\left(\sqrt{2}+5\right)(3)}=\frac{\left(3-\sqrt{5}\right)}{3}=1-\frac{\sqrt{5}}{3}$$

In this example, the surd expression was present in the form of a fraction. Hence you can use the same rules used in algebraic fractions where you cancel out common expressions and simplify the final expression fully.